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Solid System with Two Massive Eccentrics on a Rough Plane: Rotational Case

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Abstract: In this paper a solid system with rotating eccentrics as internal movers is investigated. The system moves with dry friction. Massive eccentric converts its rotational motion into rotation of the body (tripod). The spinning of the tripod with the pendulum as internal mover is considered in mathematical model and experiment; the resulting system is presented in a convenient form for further investigation by numerical methods.

Keywords: Mechanical Systems, Robots, Mechanisms, Mathematical Models, Friction, Movement.

1. INTRODUCTION

There is growing interest in mechanical systems moving due to internal movement of masses without using external movers, such as wheels, legs or tracks. A new class of mechanisms (robots, mechatronic systems) based on a new type of movement is developed by scientists and engineers. Let us consider some examples of such systems in terms of design.

One of the first papers to study dynamic systems with moving internal masses is the paper of Chernous'ko (2005). The considered mechanical system consists of a solid body that moves horizontally along a straight line and an inner body that moves relative to the solid body along an axis parallel to the axis of motion of the body. The same design is used in the paper of Chernous'ko (2006) for more detailed analysis. And the continuation of the investigation of this mechanism is given in the article of Chernous'ko (2008), but we will not go into details, leaving it to the reader's discretion. Nevertheless, this mechanical design also attracts Fang et al. (2011) and Bolotnik et al. (2012). So we can confidently say that the system of this design is well studied.

In the work of Bolotnik (2006) the design of a mechanical system differs from the design in the paper of Chernous'ko (2005) in that the internal mass moves not only along the horizontal line, but also along the vertical, so the inner mass moves in a vertical plane.

In the work of Chernous'ko et al. (2013) the design of the mechanical system is as follows: an articulated mechanical system consists of a rigid body (the main body) and two identical rigid links, attached to the main body by revolute joints. The system moves in a resistive medium in a horizontal plane.

In the paper of Bizyaev et al (2014) a spherical shell with a moving rigid body inside is investigated. Internal body is fixed inside the shell by means of two different mechanisms.

Paper of Ivanov (2014) deals with the motion of a cubic rigid body (so called M-block) with internal rotor. The M-block motion is caused by a sudden brake of the rotor, leading to a jump or rolling of the body.

Sakharov (2015) considers a rigid body, which is a hollow parallelepiped. Two material points move inside the body along the sides of the mechanism.

In our previous work (Semendyaev et al. (2016)) we consider solid system consisting of a rigid body with two internal masses (in the form of eccentrics – unbalanced massive disks). Eccentrics rotate with variable angular velocity and have no contact with external medium. The body has a contact with a horizontal rough surface with dry friction and moves short steps (or slides) in plane due to rotating eccentrics.

Previously (Semendyaev et al. (2016)), the translational case of motion of the mechanical system is investigated. In this paper the rotational case of motion of the rigid body (in the form of tripod) caused by rotation of the transverse eccentric is considered.

2. MECHANISM

The model of solid system with two unbalanced disks is constructed in Moscow Institute of Physics and Technology at the Chair of Theoretical Mechanics.

The solid system is designed in SolidWorks program – Fig. 1. From a practical point of view, it is decided to use open-source building platform (Makeblock, mechanical parts and motors). Assembling of the mechanism leads to the following result – Fig. 2. The microcontroller (Arduino) controls the

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movement of eccentrics. DC motors are used as electric motors. Rigid body in the form of tripod stays on three legs with rounded plastic foots. When the transverse eccentric rotates, the body of the tripod rotates, keeping one of the legs immovable.

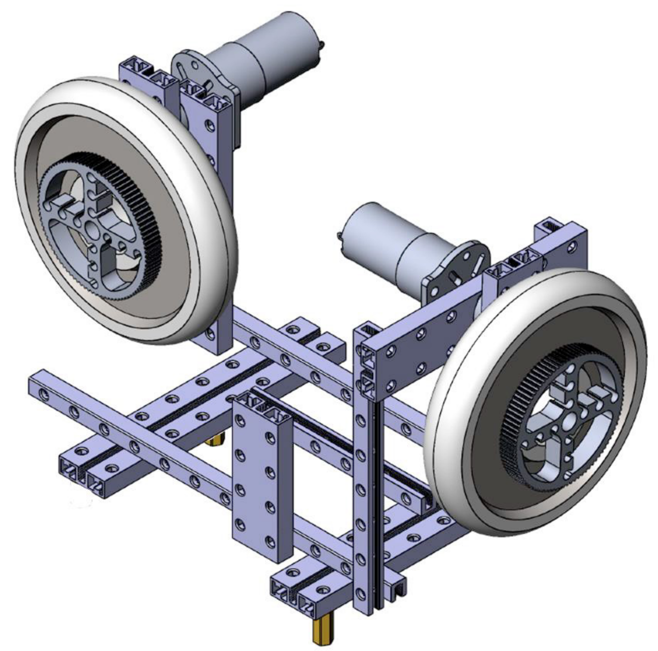

Fig. 1. CAD model of the mechanism.

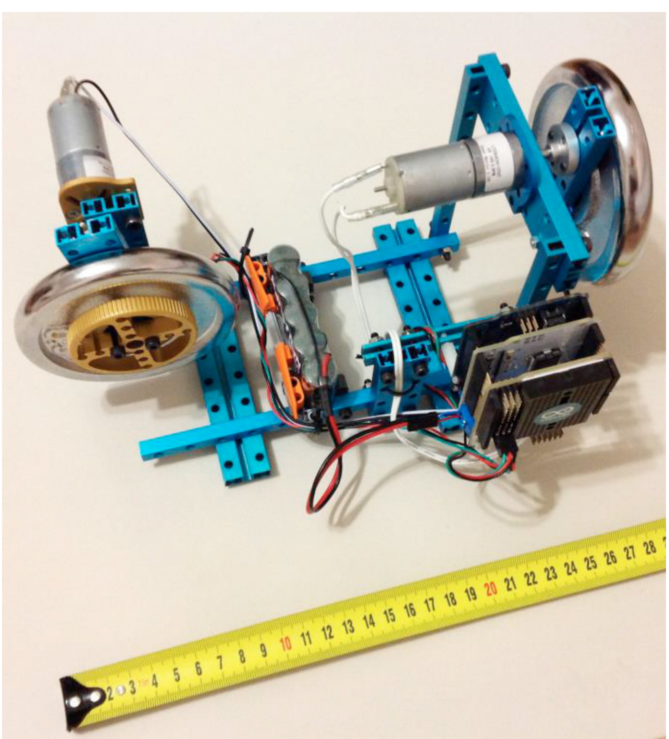

Fig. 2. Assembled mechanism.

One of the disks is placed along the longitudinal axis of symmetry, another one – in transversal position. The main parameters of the model: total weight – 1.825 kg, weight of each disk – 0.5 kg, average angular speed of the disks – 13 rad/s, eccentricity of longitudinal disk – 0.035 m, eccentricity of transversal disk – 0.025 m, the longitudinal distance between the end points of contact with the surface – 0.131 m, the transversal distance between the end points of contact with the surface – 0.113 m, coefficient of dry friction – 0.3.

3. TRANSLATIONAL CASE

In the previous paper (Semendyaev et al., 2016) the expression is obtained for the translational case when the robot moves forward or backward due to the rotation of the longitudinal eccentric and where the basic calculations are given, but here we give only the final result in order to have an idea of the system of equations. For the translational case of movement of the robot body, the equation of motion can be represented in the form (if $\upsilon \neq 0$):

$$\dot{\upsilon} = \Delta_1/\Delta, \quad \ddot{\varphi} = \Delta_2/\Delta,$$

where for $\Delta, \Delta_1, \Delta_2$ we have:

$$\Delta = Mml^2 + m^2l^2\sin^2\varphi - km^2l^2\sin\varphi\cos\varphi\frac{\upsilon}{|\upsilon|},$$

$$\Delta_1 = \left(-k\left(M+m\right)g\frac{\upsilon}{|\upsilon|} + lm\dot{\varphi}^2\left(\sin\varphi - k\cos\varphi\frac{\upsilon}{|\upsilon|}\right)\right)ml^2 -$$

$$-lm\left(\cos\varphi + k\sin\varphi\frac{\upsilon}{|\upsilon|}\right)\left(A - B|\dot{\varphi}| - mlg\sin\varphi\right),$$

$$\Delta_2 = \left(M+m\right)\left(A - B|\dot{\varphi}| - mlg\sin\varphi\right) +$$

$$+\left(k\left(M+m\right)g\frac{\upsilon}{|\upsilon|} - lm\dot{\varphi}^2\left(\sin\varphi - k\cos\varphi\frac{\upsilon}{|\upsilon|}\right)\right)ml\cos\varphi,$$

where υ – translational velocity; φ – angle between pendulum and vertical; M – mass of the support (tripod); m and l – mass and length of the pendulum; k – coefficient of friction; g – free fall acceleration; A, B – electric motor constants $\left(A, B > 0\right)$.

Then the system (taking into account also the case $\upsilon = 0$) is solved numerically in MatLab program. The results of experiments with the use of tracking are given in our article (Semendyaev et al, 2016).

4. ROTATIONAL CASE: THEORY

For consideration of our system it is convenient to divide the system into two components interacted with each other – a tripod and a pendulum.

Now let us consider the case when our mechanism rotates around a vertical axis passing through the center of mass C of the tripod. Since C does not move (the motion under consideration is a pure rotation), acceleration of C is zero. That is why we can write the balance equation for the tripod:

$$\bar{F}_C + \bar{N}_C + \bar{F}_A + \bar{F}_B + \bar{N}_A + \bar{N}_B + M\bar{g} + \bar{R}_O = 0,$$

where $\bar{F}_A, \bar{F}_B, \bar{F}_C$ – friction forces in points A, B, C; $\bar{N}_A, \bar{N}_B, \bar{N}_C$ – normal reaction in points A, B, C; $M\bar{g}$ – gravity force; $\bar{R}_O$ – reaction in point O, where the tripod is connected with a pendulum through a cylindrical hinge (see Fig. 3). In the axes $\xi\eta\zeta$ the equation looks like:

$$\begin{bmatrix} F_{C\xi} \\ F_{C\eta} \\ N_C \end{bmatrix} + \begin{bmatrix} (F_A - F_B)\sin\alpha \\ -(F_A + F_B)\cos\alpha \\ N_A + N_B - Mg \end{bmatrix} + \begin{bmatrix} -R_{O\xi} \\ -R_{O\eta} \\ -R_{O\zeta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad (1)$$

where $\alpha = const$ – angle between $C\xi$ and CA (also between $C\xi$ and CB).

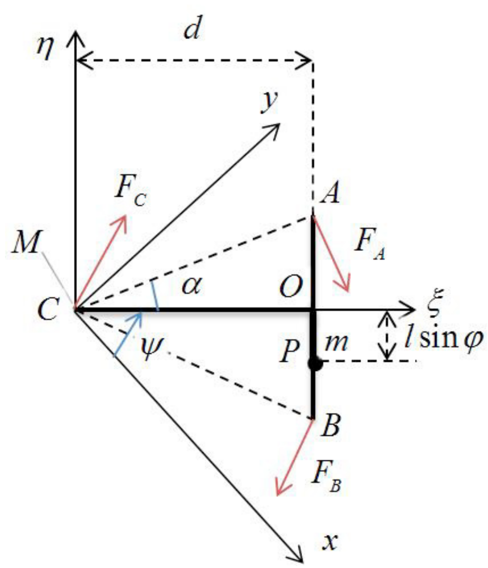

Fig. 3. Tripod with a pendulum. View from above.

Now let us write for an angular momentum of tripod:

$$\frac{d\bar{K}_C^{tripod}}{dt} = \overline{CA} \times (\bar{N}_A + \bar{F}_A) + \overline{CB} \times (\bar{N}_B + \bar{F}_B) + \bar{\mu} + \overline{CO} \times \bar{R}_O,$$

where $\bar{K}_C^{tripod} = J\dot{\psi}\bar{e}_\zeta$ – angular momentum of tripod with respect to the point C; $\overline{CA}, \overline{CB}, \overline{CO}$ – vectors from C to A, B, O; $\bar{\mu}$ – moment of reaction to the electric motor, rotating pendulum.

In projections on $C\xi\eta\zeta$ we have:

$$\begin{bmatrix} 0 \\ 0 \\ J\ddot{\psi} \end{bmatrix} = \begin{bmatrix} (N_A - N_B)d\tan\alpha + \mu + R_{O\eta}h \\ -(N_A + N_B)d + R_{O\zeta}d - R_{O\xi}h \\ -(F_A + F_B)d(\cos\alpha + \tan\alpha\sin\alpha) - R_{O\eta}d \end{bmatrix}, (2)$$

where $J = const$ – moment of inertia of the tripod relative to the axis $C\zeta$; ψ – angle of rotation of the tripod; $d = const$ – length of the tripod along the axis $C\xi$; $h = const$ – height of point O above surface.

We denote the constant that determines the geometry of the tripod, as follows:

$$\Lambda = \cos\alpha + \tan\alpha\sin\alpha = const.$$

Let us write down the equation for momentum of pendulum in a non-inertial frame of reference $C\xi\eta\zeta$ rotating with the tripod:

$$\frac{d\bar{p}_{rel}^{pendulum}}{dt} = \bar{R}^{ext} + \bar{R}^{in1} + \bar{R}^{in2},$$

where $\bar{p}_{rel}^{pendulum}$ – momentum of pendulum in non-inertial frame; $\bar{R}^{ext}$ – the sum of the external forces acting on the pendulum, $\bar{R}^{in1}, \bar{R}^{in2}$ – forces of inertia.

For the momentum of pendulum with the mass m and the length l we have:

$$\bar{p}_{rel}^{pendulum} = m\bar{V}_P^{rel} = m\begin{bmatrix} 0 \\ -\dot{\varphi}l\cos\varphi \\ \dot{\varphi}l\sin\varphi \end{bmatrix},$$

where φ – angle between the pendulum and the vertical ζ.

For external forces:

$$\bar{R}^{ext} = \bar{R}_O + m\bar{g} = \begin{bmatrix} R_{O\xi} \\ R_{O\eta} \\ R_{O\zeta} - mg \end{bmatrix},$$

where $\bar{R}_O$ here for pendulum is opposite to the $\bar{R}_O$ in the equations for tripod.

For inertia force $\bar{R}^{in1}$ we have:

$$\bar{R}^{in1} = -m\left(\ddot{\psi}\bar{e}_\zeta \times \overline{CP} + \dot{\psi}\bar{e}_\zeta \times \left(\dot{\psi}\bar{e}_\zeta \times \overline{CP}\right)\right) = = m\begin{bmatrix} \dot{\psi}^2 d - \ddot{\psi}l\sin\varphi \\ -\dot{\psi}^2 l\sin\varphi - \ddot{\psi}d \\ 0 \end{bmatrix},$$

and for inertia force $\bar{R}^{in2}$:

$$\bar{R}^{in2} = -2m\dot{\psi}\bar{e}_\zeta \times \bar{V}_P^{rel} = m\begin{bmatrix} -2\dot{\psi}\dot{\varphi}l\cos\varphi \\ 0 \\ 0 \end{bmatrix}.$$

So finally for the relative momentum of pendulum after calculating derivatives we have:

$$m\begin{bmatrix} 0 \\ -\ddot{\varphi}l\cos\varphi + \dot{\varphi}^2 l\sin\varphi \\ \ddot{\varphi}l\sin\varphi + \dot{\varphi}^2 l\cos\varphi \end{bmatrix} =$$

$$=\begin{bmatrix} R_{O\xi}+m\left(\dot{\psi}^2 d-\ddot{\psi} l\sin\varphi-2\dot{\psi}\dot{\varphi} l\cos\varphi\right) \\ R_{O\eta}-m\left(\dot{\psi}^2 l\sin\varphi+\ddot{\psi} d\right) \\ R_{O\zeta}-mg \end{bmatrix}.$$

From here we can express reaction in point O:

$$\begin{aligned} R_{O\xi}&=m\left(-\dot{\psi}^2 d+\ddot{\psi} l\sin\varphi+2\dot{\psi}\dot{\varphi} l\cos\varphi\right),\\ R_{O\eta}&=m\left(-\ddot{\varphi} l\cos\varphi+\dot{\varphi}^2 l\sin\varphi+\dot{\psi}^2 l\sin\varphi+\ddot{\psi} d\right),\\ R_{O\zeta}&=mg+m\left(\ddot{\varphi} l\sin\varphi+\dot{\varphi}^2 l\cos\varphi\right). \end{aligned} \quad (3)$$

If we put these expressions in (2), we obtain N_A, N_B, F_A+F_B.

We can also find a condition when there is no spinning of tripod:

$$\begin{cases} F_A+F_B\le k\left(N_A+N_B\right),\\ \ddot{\psi}=\dot{\psi}=0, \end{cases} \quad (4)$$

where k – coefficient of dry friction.

Taking into account this condition, from (3) we have:

$$\begin{aligned} R_{O\xi}&=0,\\ R_{O\eta}&=m\left(-\ddot{\varphi} l\cos\varphi+\dot{\varphi}^2 l\sin\varphi\right),\\ R_{O\zeta}&=mg+m\left(\ddot{\varphi} l\sin\varphi+\dot{\varphi}^2 l\cos\varphi\right). \end{aligned}$$

Let us put it in (2) and we obtain:

$$\begin{aligned} N_A+N_B&=R_{O\zeta}=mg+m\left(\ddot{\varphi} l\sin\varphi+\dot{\varphi}^2 l\cos\varphi\right),\\ F_A+F_B&=\frac{-R_{O\eta}}{\Lambda}=\frac{m\left(\ddot{\varphi} l\cos\varphi-\dot{\varphi}^2 l\sin\varphi\right)}{\Lambda}. \end{aligned}$$

So, for (4) we have:

$$\begin{cases} \dfrac{\ddot{\varphi} l\cos\varphi-\dot{\varphi}^2 l\sin\varphi}{\Lambda}\le k\left(g+\ddot{\varphi} l\sin\varphi+\dot{\varphi}^2 l\cos\varphi\right),\\ \ddot{\psi}=\dot{\psi}=0, \end{cases} \quad (5)$$

If this condition is not satisfied, then we have the rotation of tripod, and $F_A+F_B=k\left(N_A+N_B\right)$. Then, from (2) we obtain:

$$N_A=-\frac{1}{2}\left(\frac{\mu+R_{O\eta}h}{d\tan\alpha}-R_{O\zeta}+R_{O\xi}\frac{h}{d}\right),$$

$$N_B=\frac{1}{2}\left(\frac{\mu+R_{O\eta}h}{d\tan\alpha}+R_{O\zeta}-R_{O\xi}\frac{h}{d}\right),$$

where $R_{O\xi}, R_{O\eta}, R_{O\zeta}$ should be taken from (3).

Since $F_A+F_B=k\left(N_A+N_B\right)$ (when the tripod rotates) and:

$$N_A+N_B=R_{O\zeta}-R_{O\xi}\frac{h}{d},$$

then the third line in equation (2) will be written in the form:

$$\begin{aligned} J\ddot{\psi}&=-k\left(R_{O\zeta}-R_{O\xi}\frac{h}{d}\right)\Lambda d-R_{O\eta}d=\\ &=-km\left(g+\ddot{\varphi} l\sin\varphi+\dot{\varphi}^2 l\cos\varphi\right)\Lambda d-\\ &-km\left(\dot{\psi}^2 d-\ddot{\psi} l\sin\varphi-2\dot{\psi}\dot{\varphi} l\cos\varphi\right)\Lambda h+\\ &+m\left(\ddot{\varphi} l\cos\varphi-\dot{\varphi}^2 l\sin\varphi-\dot{\psi}^2 l\sin\varphi-\ddot{\psi} d\right)d. \end{aligned} \quad (6)$$

Later we shall return to this equation.

Let us write equation for angular momentum of the pendulum in non-inertial reference frame $O\xi\eta\zeta$:

$$\frac{d\bar{K}_{O\,rel}^{pendulum}}{dt}=\bar{M}_O^{ext}+\bar{M}_O^{in1}+\bar{M}_O^{in2}, \quad (7)$$

where $\bar{K}_{O\,rel}^{pendulum}=-ml^2\dot{\varphi}\bar{e}_\xi$ – relative angular momentum of the pendulum; $\bar{M}_O^{ext}$ – moment of external forces; $\bar{M}_O^{in1}, \bar{M}_O^{in2}$ – moments of inertia forces. All these moments are taken with respect to the point O.

For the derivative in left we have:

$$\frac{d\bar{K}_{O\,rel}^{pendulum}}{dt}=-ml^2\ddot{\varphi}\bar{e}_\xi.$$

For $\bar{M}_O^{ext}$ we have:

$$\bar{M}_O^{ext}=\overline{OP}\times m\bar{g}+\bar{\mu}=\left(mgl\sin\varphi-\mu\right)\bar{e}_\xi,$$

where $\bar{\mu}$ – moment of the electric motor rotating pendulum – here it is opposite to the $\bar{\mu}$ in equation for tripod (2).

For $\bar{M}_O^{in1}$ we have:

$$\bar{M}_O^{in1}=\overline{OP}\times\bar{R}^{in1}= =ml\begin{bmatrix} -\cos\varphi\left(\dot{\psi}^2 l\sin\varphi+\ddot{\psi} d\right) \\ -\cos\varphi\left(\dot{\psi}^2 d-\ddot{\psi} l\sin\varphi\right) \\ \sin\varphi\left(\dot{\psi}^2 d-\ddot{\psi} l\sin\varphi\right) \end{bmatrix}.$$

For $\bar{M}_O^{in2}$ we have:

$$\bar{M}_O^{in2}=\overline{OP}\times\bar{R}^{in2}= =ml^2\begin{bmatrix} 0 \\ 2\dot{\psi}\dot{\varphi}\cos\varphi \\ -2\dot{\psi}\dot{\varphi}\sin\varphi\cos\varphi \end{bmatrix}.$$

Substituting these expressions into equation (6), we obtain:

$$\begin{bmatrix} -ml^2\ddot{\varphi} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} mgl\sin\varphi - \mu - ml\cos\varphi\left(\dot{\psi}^2 l\sin\varphi + \ddot{\psi}d\right) \\ -ml\cos\varphi\left(\dot{\psi}^2 d - \ddot{\psi}l\sin\varphi\right) + 2ml^2\dot{\psi}\dot{\varphi}\cos\varphi \\ ml\sin\varphi\left(\dot{\psi}^2 d - \ddot{\psi}l\sin\varphi\right) - 2ml^2\dot{\psi}\dot{\varphi}\sin\varphi\cos\varphi \end{bmatrix}.$$

Here, from the second and the third line we can write:

$$\ddot{\psi}l\sin\varphi = \dot{\psi}^2 d - 2\dot{\psi}\dot{\varphi}l\cos\varphi.$$

And the first line we can rewrite in a form:

$$\ddot{\varphi} = \ddot{\psi}\frac{d}{l}\cos\varphi + \frac{\mu}{ml^2} - \frac{g}{l}\sin\varphi + \dot{\psi}^2\sin\varphi\cos\varphi. \quad (8)$$

We can substitute these expressions in equation (6) and obtain the following:

$$\begin{aligned} &\left(J + kmd^2\Lambda\sin\varphi\cos\varphi + md^2\sin^2\varphi\right)\ddot{\psi} = \\ &= -kmd\Lambda\left(g\cos^2\varphi + \dot{\varphi}^2 l\cos\varphi\right) - \\ &-kmd\Lambda\frac{\sin\varphi}{l}\left(\frac{\mu}{m} + \dot{\psi}^2 l^2\sin\varphi\cos\varphi\right) - \\ &-md\left(-\frac{\cos\varphi}{l}\left(\frac{\mu}{m} - gl\sin\varphi\right) + \dot{\psi}^2 l\sin^3\varphi + \dot{\varphi}^2 l\sin\varphi\right). \end{aligned}$$

Assuming that for any angle φ we have:

$$J + kmd^2\Lambda\sin\varphi\cos\varphi + md^2\sin^2\varphi > 0,$$

we can write:

$$\ddot{\psi} = \frac{\mathrm{E}\left(\varphi,\dot{\varphi},\dot{\psi},\mu\right)}{\Phi(\varphi)}, \quad (9)$$

where we denote the functions:

$$\Phi(\varphi) = J + kmd^2\Lambda\sin\varphi\cos\varphi + md^2\sin^2\varphi,$$

$$\begin{aligned} &\mathrm{E}\left(\varphi,\dot{\varphi},\dot{\psi},\mu\right) = -kmd\Lambda\left(g\cos^2\varphi + \dot{\varphi}^2 l\cos\varphi\right) - \\ &-kmd\Lambda\frac{\sin\varphi}{l}\left(\frac{\mu}{m} + \dot{\psi}^2 l^2\sin\varphi\cos\varphi\right) - \\ &-md\left(-\frac{\cos\varphi}{l}\left(\frac{\mu}{m} - gl\sin\varphi\right) + \dot{\psi}^2 l\sin^3\varphi + \dot{\varphi}^2 l\sin\varphi\right). \end{aligned}$$

Substituting (9) in (8), we obtain:

$$\ddot{\varphi} = \frac{\mathrm{E}\left(\varphi,\dot{\varphi},\dot{\psi},\mu\right)}{\Phi(\varphi)}\frac{d}{l}\cos\varphi + \frac{\mu}{ml^2} - \frac{g}{l}\sin\varphi + \dot{\psi}^2\sin\varphi\cos\varphi,$$

or, if we denote new function:

$$\begin{aligned} &\Gamma\left(\varphi,\dot{\varphi},\dot{\psi},\mu\right) = \\ &= \frac{\mathrm{E}\left(\varphi,\dot{\varphi},\dot{\psi}\right)}{\Phi(\varphi)}\frac{d}{l}\cos\varphi + \frac{\mu}{ml^2} - \frac{g}{l}\sin\varphi + \dot{\psi}^2\sin\varphi\cos\varphi, \end{aligned}$$

we can write:

$$\ddot{\varphi} = \Gamma\left(\varphi,\dot{\varphi},\dot{\psi},\mu\right).$$

So, finally we have the system:

$$\begin{cases} \ddot{\psi} = \dfrac{\mathrm{E}\left(\varphi,\dot{\varphi},\dot{\psi},\mu\right)}{\Phi(\varphi)}, \quad \Phi(\varphi) > 0, \\ \ddot{\varphi} = \Gamma\left(\varphi,\dot{\varphi},\dot{\psi},\mu\right). \end{cases} \quad (10)$$

The dynamics of the electric motor can be described by the equation:

$$\tau\frac{d\mu}{dt} + \mu = b_1 U - b_2\dot{\varphi}, \quad (11)$$

where μ – torque of electric motor; τ, b_1, b_2 – electric motor constants; U – voltage applied to the motor; and also:

$$\tau, b_1, b_2, U, \dot{\varphi} > 0.$$

In all calculations, we assume that point C is fixed. If we consider equation (1), then for the force of friction and the normal reaction in point C we have:

$$\begin{aligned} F_{C\xi} &= R_{O\xi} - \left(F_A - F_B\right)\sin\alpha, \\ F_{C\eta} &= R_{O\eta} + \left(F_A + F_B\right)\cos\alpha, \quad (12) \\ N_C &= R_{O\zeta} - \left(N_A + N_B\right) + Mg. \end{aligned}$$

Point C does not move when the tripod rotates, if:

$$F_C \le kN_C.$$

Hence, we should write:

$$\left(F_{C\xi}\right)^2 + \left(F_{C\eta}\right)^2 \le \left(kN_C\right)^2$$

Or, if we put $F_A = kN_A, F_B = kN_B$ in (12) when tripod rotates with fixed point C, then:

$$\begin{aligned} &\left(R_{O\xi} + k\left(\frac{\mu}{d} + R_{O\eta}\frac{h}{d}\right)\cos\alpha\right)^2 + \\ &+\left(R_{O\eta} + k\left(R_{O\zeta} + R_{O\xi}\frac{h}{d}\right)\cos\alpha\right)^2 \le \quad (13) \\ &\le k^2\left(R_{O\xi}\frac{h}{d} + Mg\right)^2, \end{aligned}$$

where $R_{O\xi}, R_{O\eta}, R_{O\zeta}$ should be taken from (3).

So, finally we have the following. The condition, when one of the legs (point C) does not move is in (13). The condition when there is no spinning of tripod with rotation of pendulum is in (5). If the condition (5) is not satisfied, then tripod rotates and its rotation is described by equations of motion presented in (10) and electric motor's dynamics in (11).

It can be noted that under condition of a large moment of inertia of the tripod $\left(J >> md^2\right)$, the equations of motion become the equations of rotation of the pendulum under the action of electric motor torque:

$$\begin{cases} \dot{\psi} = \ddot{\psi} = 0, \\ \ddot{\varphi} = \dfrac{\mu}{ml^2} - \dfrac{g}{l}\sin\varphi, \end{cases} \quad (14)$$

where μ should be taken from (11).

It can be also noted that the resulting equations of motion in general case of rotation with fixed point are $2\pi-$periodic with respect to the angular variables ψ and φ.

4. ROTATIONAL CASE: EXPERIMENT

The experimental setup consists of the mechanism (tripod with eccentrics), a horizontal rough surface on which the mechanism is located, and a video camera located on top. The camcorder records the changes in the angle of rotation of the body by two marks on its case. Tracking of marks in the video file is done by video processing program. Fig. 4 shows the tracking result obtained during the experiment.

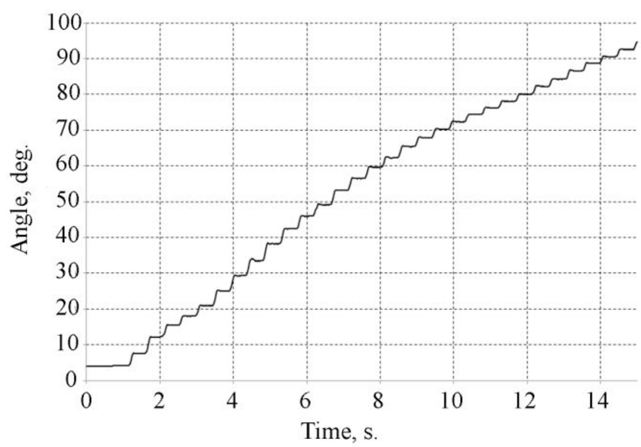

Fig. 4. Experimental result: angle of rotation of the tripod as a function of time.

As we can see, the experimental rotation of $\pi/2$ radians was done in about 15 seconds; the average angular velocity of the tripod is 0.1 rad/s. To complete one full turn of the tripod, it takes about 60 seconds, while the pendulum at the same time makes about 120 full revolutions. Note that, unlike the translational case (Semendyaev et al. (2016)), when each step forward was made with some backtracking, in the rotational case, there is almost no step back.

Fig. 4 shows a certain curvature of the midline. This can be explained by the inhomogeneity of the rough surface, where different regions have a slightly different coefficient of friction. If the curvature of the midline is not taken into account, then it can be seen that the motion is approximately periodic. This is in good agreement with the obtained equations of motion.

5. CONCLUSIONS

In this paper, the mechanical system consisting of the tripod and two eccentrics is considered. The motion of main body (tripod) depends on the motion of internal masses (eccentrics). In mathematical model of mechanical system equations for rotational motion of the tripod with one fixed point are obtained. It is shown that motion of the tripod is due to rotation of the pendulum. It can also be said that rotation of the pendulum depends on motion of the tripod. The result of experiment is also presented, showing fundamental possibility of rotational motion of the tripod, depending on rotation of the eccentrics. Although in framework of this article numerical experiments are not considered it is planned in future. It is also planned to involve averaging methods for further investigation of the system.

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